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PROBLEMS AND SOLUTIONS.

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PROBLEMS FOR SOLUTION.

[N.B. Problems containing results believed to be new, or extensions of old results are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known text-books, or results found in readily accessible sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist members of the Association with their difficulties in the solution of such problems.]

2951. Proposed by B. F. FINKEL, Drury College.

What is the average linear velocity of a point on the periphery of a locomotive driving wheel 2 feet in diameter and making 100 revolutions per minute?

2952. Proposed by R. M. MATHEWS, Wesleyan University.

Problem 2900 (*1921*, 277) suggests the following: Derive formulas for the products:

$$\cos\theta\cos\frac{1}{2}\theta\cos\frac{1}{2^2}\theta\cdots\cos\frac{1}{2^n}\theta,$$

$$\sin\theta\sin\frac{1}{2}\theta\sin\frac{1}{2^2}\theta\cdots\sin\frac{1}{2^n}\theta,$$

and

$$\tan\theta\tan\frac{1}{2}\theta\tan\frac{1}{2^2}\theta\cdots\tan\frac{1}{2^n}\theta.$$

2953. Proposed by C. F. GUMMER, Queen's University.

An algebraic equation being known to have exactly r real roots, all simple, is it possible to find an equation of degree $r - 1$ whose roots separate those of the given equation, and whose coefficients are rational functions of the coefficients of the given equation?

2954. Proposed by C. N. MILLS, Heidelberg University.

A machine-gun is placed on an armored train which is moving with a velocity of v feet per second along a straight horizontal track. The muzzle velocity of the bullets is v feet per second. Find the greatest range, (1) in front of the train and (2) behind the train.

2955. Proposed by the late L. G. WELD.

Find the proportions of an anchor ring such that its section by a plane parallel to its axis and tangent to its inner circle (circle of the gorge) shall be a lemniscate.

2956. Proposed by W. D. LAMBERT, U. S. Coast and Geodetic Survey.

A particle is constrained to move on the outer surface of a sphere whose radius equals the mean radius of the earth and which is rotating with the same angular velocity, ω, as the earth. The particle is acted upon by a force equal to $m\omega^2 \sin\phi\cos\phi$, which is directed to the nearer pole, m being the mass of the particle and ϕ the latitude. This force balances the equatorward tendency of a particle initially at rest relative to the surface; on the earth its place is supplied by the slight departure from sphericity. Show that for small velocities relative to the rotating surface the particle when acted upon by no forces except the one mentioned and by the constraint, will describe a curve differing slightly from a small circle on the sphere.

Discuss the form of the curve when the velocity relative to the surface is increased. Would it be possible to lay out a railroad track from New York to Chicago such that its departure from the great circle connecting those points would counterbalance the pressure on the rails due to the earth's rotation?

2957. Proposed by J. L. WALSH, Harvard University.

The envelope of the circles of curvature of a curve is, in part at least, the curve itself. What further curves, if any, are parts of this envelope?

2958. Proposed by R. P. BAKER, University of Iowa.

Over a frictionless pulley a weightless cord sustains at one end a mass M, while the other end is wound on the axle of a wheel of mass M and moment of inertia N. At the zero of time the wheel revolves with angular velocity ω and tends to wind up the cord. Describe the motion neglecting friction.

SOLUTIONS

2813 [1920, 81]. Proposed by PAUL CAPRON, U. S. Naval Academy.

An ellipse having the major-axis $2a$ and the eccentricity ϵ, is revolved first about its major axis, forming a prolate spheroid, then about its minor axis forming an oblate spheroid. Show that the surfaces of these spheroids are, respectively,

$$2\pi a^2(1/\epsilon)(\sqrt{1-\epsilon^2}\sin^{-1}\epsilon + 1)$$

and

$$2\pi a^2\left[2 + (1/\epsilon)(1-\epsilon^2)\log\left(\frac{1+\epsilon}{1-\epsilon}\right)\right].$$

Solution by H. S. Uhler, Yale University.

Case I. Prolate spheroid.

Let the equation of the ellipse be

$$b^2x^2 + a^2y^2 - a^2b^2 = 0. \tag{1}$$

We may take as element of surface the lateral area of the frustum of a right circular cone having the following specifications: planes of bases normal to the x-axis and at distances x and $x + dx$ from the origin, and generatrix of lateral surface tangent to the revolving ellipse at the point: (x, y). Slant height $\equiv dt$.

Then

$$dt = dx\sqrt{1 + (dy/dx)^2}.$$

From (1)

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y};$$

hence

$$dt = \frac{bdx}{a^2y}\sqrt{a^4 - (a^2 - b^2)x^2}.$$

Accordingly, the element of surface, $d\sigma$, is given by

$$d\sigma = 2\pi y dt = \frac{2\pi b dx}{a^2}\sqrt{a^4 - (a^2 - b^2)x^2},$$

and the required area of the prolate spheroid,

$$A_1 = \frac{4\pi b}{a^2}\int_0^a dx\sqrt{a^4 - (a^2 - b^2)x^2}. \tag{2}$$

By making use of the standard formula,

$$\int d\varphi\sqrt{n^2 - \varphi^2} = \frac{1}{2}\varphi\sqrt{n^2 - \varphi^2} + \frac{1}{2}n^2\sin^{-1}\frac{\varphi}{n},$$

it is merely a matter of simple reductions to change equation (2) to the following form,

$$A_1 = 2\pi b\left[b + \frac{a^2}{\sqrt{a^2 - b^2}}\sin^{-1}\left(\frac{\sqrt{a^2 - b^2}}{a}\right)\right]. \tag{3}$$

Now, by the definition of e, $b^2 = a^2(1 - e^2)$; so that, by elimination of b, equation (3) may be expressed as the following function of a and e:

$$A_1 = 2\pi a^2(e^{-1}\sqrt{1 - e^2}\sin^{-1}e + 1 - e^2). \tag{4}$$

Case II. Oblate spheroid.

Proceeding as above, it will be found that

$$A_2 = \frac{4\pi a}{b^2}\int_0^b dy\sqrt{(a^2 - b^2)y^2 + b^4}.$$